

# New Instantons in $AdS_4/CFT_3$ from D4-Branes Wrapping Some of $CP^3$

M. Naghdi \*

*Department of Physics, Faculty of Basic Sciences,  
University of Ilam, Ilam, West of Iran.*

February 4, 2013

## Abstract

With use of a six-form field-strength of ten-dimensional type IIA supergravity over  $AdS_4 \times CP^3$ , when  $S^7/S_k$  is considered as a  $S^1$  Hopf-fibration on  $CP^3$ , we earn a fully localized solution in bulk of Euclideanized  $AdS_4$ . Indeed, this object appears in external space because of wrapping of a D4(M5)-brane over some parts of respective internal spaces. Interestingly, this supersymmetry breaking  $SU(4) \times U(1)$ -singlet mode exists in already known spectrums when one use  $\mathbf{8}_c$  gravitino representation of  $SO(8)$ . To adjust the boundary theory, we should swap the original  $\mathbf{8}_s$  and  $\mathbf{8}_c$  representations for supercharges and fermions in Aharony, Bergman, Jafferis and Maldacena model. The procedure could then interpret as adding of a anti-D4(M5)-brane to the prime  $\mathcal{N} = 6$  membrane theory resulting in a  $\mathcal{N} = 0$  anti-membrane theory, while other symmetries preserve. Then, according to the well-known state-operator correspondence rules, we find a proper dual operator of the conformal dimension  $\Delta_+ = 3$  that matches to the bulk massless pseudo-scalar state. After that, by making use of some fitting ansatzs for the used matter fields, we arrive at an exact boundary solution and comment on other related issues as well.

---

\*E-Mail: m.naghdi@mail.ilam.ac.ir

# 1 Introduction

Instantons and Solitons as the well-known non-perturbative effects play many creditable roles in Mathematics and Physics especially. In the last few decades their patterns in the gauge/string dualities have become even more important by the adventure of AdS/CFT correspondence [1]. The Instantons were widely studied in framework of the famous duality of ten-dimensional (10d henceforth) type IIB string theory over  $AdS_5 \times S^5$  versus four-dimensional  $\mathcal{N} = 4$   $SU(N)$  Yang-Mills theory in [2], [3] and [4] first. Afterwards, in mid of 2008 after releasing the best so far sample of the  $AdS_4/CFT_3$  duality by Aharony, Bergman, Jafferis and Maldacena (henceforth ABJM) [5] the instanton studies, with many other efforts to discover various perspectives of the model, getting started first in [6]. Some time later, through a few studies [7], [8] and [9], we also found some new instanton vacua of the theory.

Our first solution [7] was in 11d supergravity, which reduces the field equations over the skew-whiffed  $AdS_4 \times S^7$  background to a conformally coupled scalar equation in bulk of  $AdS_4$ . By detecting the exact solutions of the equation, we analyzed their behaviors near to the boundary according to the well-known AdS/CFT correspondence rules [10]. Then to find the dual boundary operator we must have exchanged the representations  $\mathbf{8}_s$  and  $\mathbf{8}_c$  of the boundary membranes in ABJM theory. The resulting theory is then for anti-membranes. Thereafter, by deforming the boundary theory by the founded operators we arrived in exact classical solutions, which are in a sound correspondence with the bulk solutions.

The second solution [8] was in 10d type IIA supergravity over the geometric background of  $AdS_4 \times CP^3$ . Now the localized solution in bulk was a monopole-instanton. In fact, in the latter case, we had a massless  $U(1)$  gauge field in bulk of Euclideanized  $AdS_4$  ( $EAdS_4$ ) space, whose excitation induced a magnetic field on the boundary. By turning on a boundary scalar field next to the  $U(1) \times U(1)$  part of full gauge group and making use of symmetries, we found the dual boundary operator and saw how both side solutions match clearly. In [9], as well, the latter  $U(1)$  instanton in  $AdS_4$  is studied by an almost similar way besides an uplift of the exact bulk solution to the respective 11d supergravity.

In this note we continue the lines of studies to find instantons as finite actions Euclidean solutions to the equations of motion. Although the new solution is more proper to be known as an equivalent for the famous D-instanton solution of  $AdS_5/CFT_4$  duality studied in [3], [4]. We propose an ansatz for the five-form (six-form) field in the type IIA(M) supergravity version of the ABJM model while the main background geometry and fields keep unchanged. Doing so, we get a localized solution in bulk of  $EAdS_4$ . The origin of this object in the bulk is likely from the winding of added D4/M5-branes around some parts of the internal  $CP^3$  or  $S^7/Z_k$  of the complete 10d or 11d geometries. The ansatzs and solutions interestingly preserve the original symmetries but break all super-symmetries. It is indeed a pseudo-scalar and singlet of  $SU(4) \times U(1)$  isometry group arising because of regarding the prime  $S^7$  as a  $U(1)$  Hopf-fibration on  $CP^3$ . The basic motivate for the mode to be known as a pseudo-scalar is its coming from the form fields which are in terms of the internal space ingredients.

On the other hand, we see that in type IIA/M supergravity spectrums for the involved Hopf-fibration and Lens spaces (i.e.  $AdS_4 \times S^7$ ,  $AdS_4 \times S^7/Z_k$  and  $AdS_4 \times CP^3$ ) as first traced in [11],

there is a singlet uncharged pseudo-scalar in bulk, which corresponds to a marginal operator in a 3d  $\mathcal{N} = 0$  boundary CFT with the global symmetry  $SU(4)_R \times U(1)$ . The last  $U(1) \sim SO(2)$  becomes the baryonic symmetry in ABJM theory while  $R$  shows  $SU(4) \sim SO(6)$  R-symmetry of the boundary field theory that is also the isometry group of  $CP^3$ .

Now an important point is that aforesaid pseudo-scalar, which sits in representation  $\mathbf{1}_0$  of  $SU(4) \times U(1)$  exists just when gravitons are in representations  $\mathbf{8}_c$  or  $\mathbf{8}_v$  of original  $SO(8)$  while gravitons or super-symmetry charges are originally in  $\mathbf{8}_s$  of ABJM. So in a similar line with [7], to adjust the bulk and boundary solutions, we again should swap the representations  $\mathbf{c}$  and  $\mathbf{s}$  in ABJM. The resultant theory is so for anti-membranes and one then may also conclude the branes we are wrapping over the internal spaces are actually anti D/M-branes.

To find a plain counterpart boundary solution, we first note that we have a massless scalar field in the bulk and therefore the dual boundary operator should have the conformal dimension  $\Delta_{\pm} = 3, 0$ . The upper branch mode, which corresponds to the normalized bulk modes, is suitable for us in that non-normalizable solutions are not indeed corresponding to bulk fluctuations but they present coupling of some external sources to supergravity or string theory. Second, we note that various terms in  $SU(4)_R \times U(1)_b$ -invariant lagrangian of ABJM [5], [12], [13] have the right dimension of 3. Third, it is proven that deformations with marginal boundary operators are not indeed deformations of the boundary theory but there may be new states in the same theory [14]. Fourth, we may also look at the boundary operators for such modes, which are proposed for example in [15], [16] and [17].

All these together suggest what operator to deform the boundary theory with we should use. Therefore, we handle an operator of the similar type with the Fermi potential terms of the ABJM  $SU(4)_R \times U(1)_b$ -invariant lagrangian. Another alternative operator we may use to adjust the bulk/boundary solutions is the gauge parts of the mentioned lagrangian similar with the procedure of Yang-Mills instantons finding in  $\mathcal{N} = 4$   $SU(N)$  field theory. Afterwards, to match the bulk and boundary solutions according to gravity/gauge duality rules [[10], we just turn on one scalar and one fermion alongside the  $U(1) \times U(1)$  part of the full quiver gauge group of the model.

Organizing this paper is as follows. In Section 2, we give a brief necessary review of the field theory and gravity side of the ABJM model. For gravity side, we start from 11d supergravity and concisely arrive in 10d type IIA supergravity of ABJM. For Field theory side, we present the standard lagrangian of the model alongside the needed symbols. In Section 3, gravity side ansatzs, equations of motion to be satisfied, solutions and their associated interpretations and discussions inspect. The spectrums of involved supergravities and how to arrive at our wished representation is also addressed. There we also evaluate the action and added brane charges based on the founded solutions and discuss briefly on uplifting ansatz and solution to 11d supergravity. Section 4 is allocated to study and find the dual field theory solutions and counterparts. There, we review the bulk-boundary correspondence rules for the case, set up the dual boundary operator and present a clear solution next to matching the bulk and boundary facts with a confirmation that way is right. Section 5 includes summary, comments on super-symmetry and stability and some other related issues.

## 2 A Brief of the Gauge/Gravity of the ABJM Model

The Aharony, Bergman, Jafferis, Maldacena (ABJM) model [5] is the best so far known version for  $AdS_4/CFT_3$  correspondence. It states on near horizon limit of a stack of  $N$  coincident M2-brane probing a singularity in  $C^4/Z_k$  orbifold (which is indeed the IR limit), a three-dimensional  $U(N)_k \times U(N)_{-k}$  Chern-Simons-matter theory with level  $(k, -k)$  coupled to matter fields in the bi-fundamental representation lives. The model has an  $\mathcal{N} = 6$  supersymmetry for generic  $k$ , which enhances to  $\mathcal{N} = 8$  non-perturbatively when the Chern levels  $k$  are  $k = 1, 2$ . For the last values of  $k$ , the theory describes M2-branes in flat space and  $R^8/Z_2$ , respectively. The model is conjectured to have a gravitational dual description that is M-theory over  $AdS_4 \times S^7/Z_k$  and under some conditions type IIA string theory over  $AdS_4 \times CP^3$  we describe more bellow.

### 2.1 The Gravity Side of the Model

To arrive in the near horizon limit of the model, one can start from the  $AdS_4 \times S^7$  solution of 11d supergravity with  $\hat{N}(= kN)$  unit of the 4-form flux as follows:

$$ds_{ABJM(M)}^2 = \frac{R^2}{4} ds_{AdS_4}^2 + R^2 ds_{S^7}^2 \quad (2.1)$$

$$G_4^{(0)} \approx \hat{N} \mathcal{E}_{AdS_4} \quad (2.2)$$

where  $R$ ,  $\hat{N}$  and  $\mathcal{E}_{AdS_4}$  are the curvature radius of the 11d target-space, the initial number of flux quanta and unit volume-form of  $AdS_4$ , respectively. The  $AdS_4$  metric in Poincare upper-half plane coordinate we use here with the Euclidean signature is

$$ds_{EAdS_4}^2 = \frac{L^2}{u^2} (du^2 + dx_i dx_i), \quad i = 1, 2, 3 \quad (2.3)$$

with noting that  $2L = R = R_7 = 2R_{AdS}$ .

One always can parameterize the transverse space to M2-branes through four complex coordinates  $X_I$  ( $I = 1, 2, 3, 4$ ), which are the needed coordinates (scalars) to embed in the round seven-sphere  $S^7$  as  $\sum_{I=1}^4 |X_I|^2 = 1$ . Now by considering  $S^7$  as a  $S^1$  fibration over  $CP^3$ , one can write

$$ds_{S^7}^2 = ds_{CP^3}^2 + (d\phi + \omega)^2, \quad (2.4)$$

where  $\omega$  is a topologically nontrivial one-form on  $CP^3$  (which is dual to the Reeb killing vector  $\partial_{\phi}$ ),  $\phi$  is the  $U(1)$  fiber coordinate with a period of  $2\pi$ , and the unit-radius metric of  $CP^3$

with a specific six real coordinates reads

$$ds_{CP^3}^2 = d\xi^2 + \cos^2\xi \sin^2\xi \left( d\psi + \frac{1}{2}\cos\theta_1 d\varphi_1 + \frac{1}{2}\cos\theta_2 d\varphi_2 \right)^2 + \frac{1}{4}\cos^2\xi (d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) + \frac{1}{4}\sin^2\xi (d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2) \quad (2.5)$$

and as well

$$\omega = \frac{1}{2}((\cos^2\xi - \sin^2\xi)d\psi + \cos^2\xi \cos\theta_1 d\varphi_1 + \sin^2\xi \cos\theta_2 d\varphi_2) \quad (2.6)$$

where  $0 \leq \xi \leq \pi/2$ ,  $0 \leq \chi_s, \varphi_s, \dot{\varphi}, \psi \leq 2\pi$ ,  $0 \leq \theta_s \leq \pi$ ,  $s = 1, 2$ .

Here the operation of  $Z_k$  quotient (orbifold) of  $C^4$  on the four complex scalar coordinates is as  $X_I \rightarrow e^{i2\pi/k} X_I$ . Then in order to have  $N$  unit of the 4-form flux on the quotient space, one should take  $\dot{N} = kN$  and  $\dot{\varphi} = \varphi/k$  and so new metric reads

$$ds_{ABJM(IIA)}^2 = \tilde{R}^2 (ds_{AdS_4}^2 + 4ds_{CP^3}^2), \quad \tilde{R}^2 = \frac{R^3}{4k} = \pi \sqrt{\frac{2N}{k}} = \pi \sqrt{2\lambda} \quad (2.7)$$

in which  $\lambda \equiv N/k$  is 't Hooft effective coupling constant of the boundary theory. In interesting limit of large  $N$  and for  $\lambda \ll N^{1/5}$ , the field theory is dual to M-theory over  $AdS_4 \times S^7/Z_k$  together with  $N$  unit of the  $G_7^{(0)}$  flux on  $S^7/Z_k$ . When  $k$  grows (the limit of  $k \rightarrow \infty$  nearly), the M-theory circle shrinks and a better description for dual field theory, in limit of  $N^{1/5} \ll k \ll N$ , is type IIA string theory over  $AdS_4 \times CP^3$  with  $N$  unit of the 6-form  $F_6^{(0)}$  flux on  $CP^3$  and  $k$  unit of the 2-form  $F_2^{(0)}$  flux on  $CP^1 \subset CP^3$ . These form-fields and dilation in type IIA theory are

$$e^{2\phi} = \frac{R^3}{k^3}, \quad H_3 = dB_2 = 0, \quad F_2^{(0)} = dA_1^{(0)} = kJ, \quad F_4^{(0)} = dA_3^{(0)} = \frac{3}{8}R^3 \mathcal{E}_4 \quad (2.8)$$

where  $\mathcal{E}_4$  is the  $AdS_4$  unit volume-form and  $J(=d\omega)$  is the Kähler form on  $CP^3$ .

## 2.2 The Field Theory Side of the Model

The three-dimensional  $\mathcal{N} = 6$  Chern-Simon-matter theory of ABJM is consists of  $U(N) \times U(N)$  gauge fields with the level  $k$  and  $-k$  coupled to (anti) bi-fundamental matter fields. This theory can be constructed from theories with  $\mathcal{N} = 2$  and  $\mathcal{N} = 3$  super-symmetries, where two later cases exist for any gauge group and charge contents [5]. The  $SU(4)_R \times U(1)_b$ -invariant action of ABJM is always written as follows [12], [13]:

$$S_{ABJM} = \int d^3x \left\{ \frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \text{tr} \left( A_\mu A_\nu A_\lambda + \frac{2i}{3} A_\mu \partial_\nu A_\lambda - \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \partial_\nu \hat{A}_\lambda \right) - \text{tr} (D_\mu Y_A^\dagger D^\mu Y^A) - \text{tr} (\psi^{A\dagger} i\gamma^\mu D_\mu \psi_A) - V_{bos} - V_{ferm} \right\} \quad (2.9)$$

where the first parenthesis is the Chern-Simon term and second and third ones are kinetic terms for bosons and fermions, respectively. The Bose scalar potential and Bose-Fermi interaction terms read

$$V_{bos} = -\frac{4\pi^2}{3k^2} \text{tr} (Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger) \quad (2.10)$$

$$V_{ferm} = -\frac{2\pi i}{k} \text{tr} (Y_A^\dagger Y^A \psi^{B\dagger} \psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2Y_A^\dagger Y^B \psi^{A\dagger} \psi_B + \varepsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D - \varepsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger}) \quad (2.11)$$

respectively. Here  $A_\mu, \hat{A}_\mu$  are  $U(N) \times U(N)$  gauge fields. The matter fields  $Y^A$  and  $\psi_A$  with  $(A = 1, \dots, 4)$  are four complex scalars and four three-dimensional spinor fields that each convert in bi-fundamental representation of the quiver gauge group as  $(N, \bar{N})$ . Besides the gauge symmetry, there is  $SU(4)_R \times U(1)_b$  R-symmetry under which the scalars  $Y^A$  transform as  $\mathbf{4}_1$  and fermions  $\psi_A$  sit in  $\bar{\mathbf{4}}_{-1}$ . Meanwhile, the gauge covariant derivatives for the matter fields  $\Phi$  ( $Y^A$  or  $\psi_A$ ) and field strength for  $F_{\mu\nu}$  are

$$\begin{aligned} D_\mu \Phi &= \partial_\mu \Phi + iA_\mu \Phi - i\Phi \hat{A}_\mu, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \end{aligned} \quad (2.12)$$

respectively. Traces are taken on the gauge group  $N \times N$  matrices keeping the gauge invariant quantities by setting the normalization of  $U(N)$  as  $\text{tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$ . The conventions for metric, Clifford algebra and real gamma matrices in original Minkowski signature are as

$$\eta_{\mu\nu} = \text{diag}(-1, 1, 1), \quad \{\gamma_\mu, \gamma_\nu\} = -2\eta_{\mu\nu}, \quad \gamma^\mu = (i\sigma_2, \sigma_1, \sigma_3), \quad \varepsilon^{012} = 1 \quad (2.13)$$

where  $\sigma_{1,2,3}$  are the usual Pauli matrices. We will see a small change of relations in going to Euclidean signature. Anyhow, various perspectives of the lagrangian and involved symmetries such as  $\mathcal{N} = 1, 2$  superfield formalism of theory are studied in [18], [13] among many others.

### 3 New Instanton Solution in the Bulk of $AdS_4$

#### 3.1 The Ansatz in 10-Dimension and Preliminaries

We start with an ansatz for 6-form field strength of type IIA supergravity with making use of an established form in ABJM model as

$$\begin{aligned} A_5 &= (f \omega \wedge J^2) \Rightarrow F_6 = df \wedge \omega \wedge J^2 + f J^3, \\ &\Rightarrow F_4 = *_{10} F_6 = *_4 df \wedge *_6 (\omega \wedge J^2) + k f (*_4 \mathbf{1} \wedge *_6 J^3) \end{aligned} \quad (3.1)$$

here  $f$  is a scalar function covering whole  $AdS_4$ ,  $*_{10} \equiv *$  from now and note that all coefficients are still included in the Hodge-star.

Now with noting the background geometry and fields in the model keep unchanged, it is not difficult to check that all needed relations satisfy interestingly. Clearly, the 10d type IIA supergravity action in string frame is given by

$$S_{IIA} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} e^{-2\phi} R + \frac{1}{2\kappa^2} \int \left[ e^{-2\phi} (4d\phi \wedge *d\phi - \frac{1}{2} H_3 \wedge *H_3) - \frac{1}{2} F_2 \wedge *F_2 - \frac{1}{2} \tilde{F}_4 \wedge *\tilde{F}_4 - \frac{1}{2} B_2 \wedge F_4 \wedge F_4 \right] \quad (3.2)$$

where  $H_3 = dB_2$ ,  $F_2 = dA_1$ ,  $F_4 = dA_3$ ,  $\tilde{F}_4 = dA_3 - A_1 \wedge H_3$  and the Hodge-star operation is with respect to full 10d metric. By taking  $H_3 = 0$  same as ABJM, the formic relations to satisfy then are

$$dF_p = 0, \quad d * F_p = 0, \quad (3.3)$$

$$d * H_3 = g_s^2 (-F_2 \wedge *\tilde{F}_4 + \frac{1}{2} \tilde{F}_4 \wedge \tilde{F}_4) = 0 \quad (3.4)$$

where  $p = 2, 4$  and in the last relation use is made of the fact that dilaton is constant with  $e^{2\phi} = g_s^2$  for ABJM.

For the dilaton and metric equations satisfy, the arguments are same as our previous study [8]. Indeed, the dilaton equation satisfies automatically while the RHS of Einstein equation, on which are the energy-momentum tensors, remain to be satisfied. We of course may use the same reasonable tricks in [8] to dissolve the problem. There we noted that because the coefficient in front of related energy-momentum tensors in the RHS of the Einstein equation is  $e^{2\phi} = R^3/k^3$ , for a limit of large  $k$  that is the legality limit of type IIA version of ABJM, the added effect is negligible. Nevertheless, since the asymptotic symmetry groups on both sides remain unchanged one may argue the back-reaction, if any, is tiny for our probe approximation especially. On the other hand, as long as we are interested in behavior of the solution near to the boundary and correlation functions of dual operators, the back-reactions on the background geometry could be ignored [19].

### 3.2 Discussions on Solutions and Spectrums

One may proceed through the supersymmetry transformations for gravitinos in 10d or 11d to obtain solutions and amounts of preserved supersymmetry. But here we go through satisfying the equations of motion directly.

The ansatz (3.1) satisfies  $d * F_4 = 0$  trivially while to satisfy  $dF_4 = 0$ , the non-trivial conditions read

$$d(*_4 df) = 0, \quad d * (\omega \wedge J^2) = 0. \quad (3.5)$$

With using the relations (2.5) and (2.6), one can affirm the second relation satisfies fortunately whereas the first one, which is indeed the Laplace equation in  $EAdS_4$ , results in

$$\frac{1}{\sqrt{g}} \partial_{\bar{\mu}} (\sqrt{g} g^{\bar{\mu}\bar{\nu}} \partial_{\bar{\nu}} f) = \left[ \partial_i \partial^i + u^2 \frac{\partial}{\partial u} \frac{1}{u^2} \frac{\partial}{\partial u} \right] f(u, \vec{u}) \equiv L_4 f(u, \vec{u}) = 0 \quad (3.6)$$

where  $\dot{\mu}, \dot{\nu}, \dots$  stands for four  $AdS_4$  coordinates and we define  $\vec{u} = \vec{r} = (x_1, x_2, x_3)$ . A familiar solution to this equation is

$$f(u, \vec{u}; 0, \vec{u}_0) = c_1 + \frac{c_2 u^3}{[u^2 + (\vec{u} - \vec{u}_0)^2]^3} \quad (3.7)$$

where  $c_1, c_2, \dots$  are some constant coefficients related to the brane-instanton charges, we settle later partly. This solution is familiar in that is indeed like the Green-function for a massless scalar field propagating between the place of instanton at  $(0, \vec{u}_0)$  and another point at  $(u, \vec{u})$ . That is called the bulk to bulk propagator and for the current case that source (instanton) is on the boundary of  $AdS_4$ , it is called the boundary to bulk propagator. Actually, the solution is singular at  $u = 0$  and corresponds to a small instanton on the boundary.

On the other hand, the field equation (3.6) is for a massless scalar in  $AdS_4$  and so the conformal dimensions of dual boundary operators, according to  $\Delta_{\pm} = +\Delta \pm \sqrt{\bar{d}^2 + 4(mL)^2}/2$  for  $AdS_{d+1}$ , are  $\Delta_{\pm} = 3, 0$ . For the supergravity multiplets of the lowest mass, only the upper branch  $\Delta_+$ , which is the normalizable mode, is suitable. In limit of approaching boundary ( $u \rightarrow 0$ ), the propagator (3.7) reduces to a delta function  $\delta^{(3)}(\vec{u} - \vec{u}_0)$ . This singular point is the instanton position. In type IIB theory the instanton was of D(-1)-brane [3], [4], [20], [21]. What is that here? As we could see from the ansatz structure (3.1), it may indeed interpret as a kind of object coming from the Kaluza-Klein reduction of 11d or 10d supergravity on the related spaces in ABJM with the added fields. Indeed, it seems that because of wrapping the world-volume of added Euclideanized electric D4-brane on  $\omega \wedge J^2$  part of perfect internal space of  $CP^3$ , some fluctuations appear in stature of a scalar  $f$  in external space of  $AdS_4$ .

A remarkable point is the solution (3.7) we have is a point-like or fully localized object in 4-dimensional external space  $AdS_4$ . A counterpart to this in type IIB theory over  $AdS_5 \times S^5$  is discussed in [20] and [21] while the solution in [4] is localized in whole 10d space. Therefore, the solution here is smeared on some part of  $CP^3$  or  $S^7/Z_k$ , which is of course likely to match with solutions on the boundary field theory. Whether the current solution could be uplifted to 10d or 11d parent theories is related to a fact that truncation is consistent or not <sup>1</sup>. We comment more on this point in the last section.

Another possible solution to the Laplace equation (3.6) holds by separation the scalar function in its external variables as  $f(u, \vec{u}) = f(u)f(\vec{u})$ . Then, in general, the  $u$  part solution is a simple *exponential* function while the  $\vec{u}$  part is a *distribution equation* in three dimensions. In the simplest case and after integration on three bulk coordinates  $\vec{u}$ , which are indeed the D2(M2)-branes world-volume directions, the smeared solution versus localized solution in three bulk dimensions of  $AdS_4$ , can be written as  $f(u) = c_3 + c_4 u^3$ . So the instanton is now localized just in  $u$  direction and not all  $EAdS_4$ .

An intersecting point to say is the operator  $L_4$  in (3.6) is invariant under the conformal transformation  $x_{\dot{\mu}} \leftrightarrow \frac{x_{\dot{\mu}}}{u^2 + r^2}$  and seemingly the resultant solution goes to the last one and

---

<sup>1</sup>A Kaluza-Klein truncation is consistent if only a finite set of fields hold in it meanwhile these low-dimensional fields don't disturb the upper dimensional ones or serve as sources for them. Then, any solution to the low-dimensional theory is valid in full upper-dimensional one.



the order reversed. This transformation maps a point at infinity to origin and exchanges the boundary conditions. But obstacle is that although the metric (2.1) or (2.7) is also conformal invariant yet the new form-field  $F_6$  (or  $F_4$ ), from which the solution (3.7) arises, is not. Therefore other interesting discussions on the map being abandoned automatically!

Now the question may be whether such a bulk excitation is in the known spectrum of 11d and 10d supergravities over the involved spaces  $AdS_4 \times S^7/Z_k$  and  $AdS_4 \times CP^3$  or not? The answer is fortunately yes. But first we should note that since the object is coming from wrapping a brane completely over the internal spaces (or a form-field completely in terms of the known one-form  $\omega$  on the internal manifolds), it must be a pseudo-scalar. Interestingly, there are pseudo-scalar fluctuations in the gauged supergravity over such spaces [11].

Indeed, we note that there are three representations  $\mathbf{8}_s$ ,  $\mathbf{8}_c$ ,  $\mathbf{8}_v$  of the isometry group  $SO(8)$  of  $S^7$  for gravitino. After the Hopf-fiber reduction, only the  $U(1)$ -natural states remain in the spectrum. Moreover, in the ABJM lagrangian (2.9), supersymmetry charges (gravitons), fermions and scalars decompose under the isometry group  $SU(4) \times U(1)$  of  $CP^3 \times S^1$  as

$$\begin{aligned}\mathbf{8}_s &= \mathbf{1}_2 \oplus \mathbf{1}_{-2} \oplus \mathbf{6}_0 \\ \mathbf{8}_c &= \bar{\mathbf{4}}_{-1} \oplus \mathbf{4}_1 \\ \mathbf{8}_v &= \mathbf{4}_1 \oplus \bar{\mathbf{4}}_{-1}\end{aligned}\tag{3.8}$$

respectively. This decomposition is in fact for the gravitino of  $\mathbf{s}$  for that scalars and pseudo-scalars are in  $\mathbf{35}_{v,c}$ , while gauge bosons are in  $\mathbf{28}$  for all cases.

The thirty-five scalars and thirty-five pseudo-scalars and gauge fields from 11d gauged supergravity, decompose also as

$$\begin{aligned}\mathbf{35}_{v,c} &= \mathbf{10}_2 \oplus \bar{\mathbf{10}}_{-2} \oplus \mathbf{15}_0 \\ \mathbf{35}_s &= \mathbf{1}_0 \oplus \bar{\mathbf{1}}_4 \oplus \mathbf{1}_{-4} \oplus \bar{\mathbf{6}}_2 \oplus \mathbf{6}_{-2} \oplus \mathbf{20}_0 \\ \mathbf{28} &= \mathbf{1}_0 \oplus \bar{\mathbf{6}}_2 \oplus \mathbf{6}_{-2} \oplus \mathbf{15}_0\end{aligned}\tag{3.9}$$

respectively. For the gravitino of  $\mathbf{c}$ , the only remaining scalars (pseudo-scalars) in the massless spectrum of type IIA supergravity over  $AdS_4 \times CP^3$  sit in  $\mathbf{15}_0$  ( $\mathbf{1}_0$ ) and reversely for  $\mathbf{v}$ . Dual boundary theory is then a 3d  $\mathcal{N} = 0$  CFT theory with global symmetry  $SU(4) \times U(1)$  and two marginal operators in  $\mathbf{1}_0$  and  $\mathbf{15}_0$ . For the gravitino of  $\mathbf{s}$  the only massless scalars (pseudo-scalars) sit in  $\mathbf{15}_0$  and dual boundary theory is a 3d  $\mathcal{N} = 6$  CFT theory with global symmetry  $SU(4) \times U(1)$  and tow marginal operators just in  $\mathbf{15}_0$ .

On the other hand, we note that our ansatz in (2.9) is invariant (indeed singlet) under full  $SU(4) \times U(1)$  symmetry group. It is since  $\omega$  and therefore  $J$  are invariant under  $SU(4)$  and neutral with respect to  $U(1)$ . With the fact that we have a pseudo-scalar  $0^-$  in bulk that fascinatingly exists in a skew-whiffed (or *right representation* in language of [22]) representation [11], we lead to a fact that we are winding (anti)-branes around the internal spaces with right directions (or maybe branes around internal manifolds with reverse directions). When addressing the boundary side and state-operator correspondence and in the last section we return to the subject concisely.

### 3.3 Charges and Action

Now we try to evaluate the added (anti)branes charges. So, the electric charge of D4-brane based on the solution (3.7) with  $c_1 = 0$ , through the standard formula

$$Q_e^{D4} = \frac{1}{\sqrt{2}\kappa^2} \int *F_6 \quad (3.10)$$

obtains to be

$$Q_e^{D4} = c_5 \frac{k}{R^3} \frac{1}{\epsilon^3} \quad (3.11)$$

where  $\kappa^2 = \frac{1}{2}(2\pi)^7$ ,  $\epsilon > 0$  is a regulator small parameter [19]<sup>2</sup>, and use is made of metric of (2.7) and following identities

$$\mathcal{E}_6 = \frac{1}{8.3!} J^3, \quad *J^3 = \frac{k}{128R^3} \mathcal{E}_4, \quad *\mathcal{E}_4 = \frac{R^3}{3k} J^3 \quad (3.12)$$

One could also note that to adjust with our notation for  $F_2^{(0)}$  in (2.8), we take the mentioned unit-volume element  $\mathcal{E}_6$  for  $CP^3$  and thus we should also take  $\int_{CP^1} J = 2\pi$ .

One can similarly calculate the Euclideanized (anti) D4-brane magnetic charge, which is indeed the electric charge of its dual (anti) D2-brane. Now a more interesting point about the charge in (3.11) is that according to (2.7) it is proportional to  $\sim 1/\sqrt{\lambda}$  and so in type IIA validity limit  $\lambda \gg 1$  of ABJM, it is almost negligible. This stresses our thought about ignoring the back-reaction from the added brane on the background.

Similar to charges, the correction to the action because of added field can estimate. The relevant part of the main action (3.2) is now the fifth term. By writing in the ansatz (3.1) upon the solution (3.7) with  $c_1 = 0$  into this part of action, we have

$$\begin{aligned} S_{modi.}^{D4} = & -\frac{1}{256(2\pi)^4} \frac{k^3}{R^3} \int_{AdS_4} f^2 dVol(AdS_4) \\ & - \frac{1}{2(2\pi)^7} \int_{AdS_4} (df \wedge *_4 df) \int_{CP^3} (\omega \wedge J^2) \wedge *_6 (\omega \wedge J^2) \end{aligned} \quad (3.13)$$

where  $dVol(AdS_4) = \mathcal{E}_4$ . The singular points of integrals are at  $u = 0$  and so according to regularization discussions [19], we may again keep just the finite part of the action. That is

$$S_{modif.}^{D4} = c_6 \frac{k}{R^3} \frac{1}{\epsilon^6} \quad (3.14)$$

on the boundary at  $u = \epsilon$ . We see again that correction is a small amount.

---

<sup>2</sup>It should be mentioned that just when  $x_{\hat{\mu}} = \epsilon \neq 0$ , we have a definite charge.

### 3.4 On the Ansatz Uplift to 11-Dimension

The gravitational field spectrums, which are chiral primary on the ABJM background, are indeed projections of the original spectrum on  $AdS_4 \times S^7$  into  $Z_k$ -invariant states [5]. Indeed, the orbifold  $Z_k$  preserves the  $SU(4) \times U(1)$  isometry symmetry of full  $SO(8)$  isometry symmetry of  $S^7$  as various decompositions under  $SO(8) \rightarrow SU(4) \times U(1)$  are given in (3.8). For  $k \geq 3$ , two single supercharges in  $\mathbf{8}_s$  of the original theory are projected out and remaining symmetry is just  $\mathcal{N} = 6$ . For  $k = 1, 2$ , the supersymmetry enhances to  $\mathcal{N} = 8$  because of monopole operators non-perturbatively.

In the lens-space of  $S^7/Z_k$ , for  $k \geq 3$  the pattern is almost same as  $CP^3$ . Indeed for the skew-whiffed cases (the gravitons in  $\mathbf{8}_{v,c}$ ), the boundary theory is a 3d  $SU(4) \times U(1)$   $\mathcal{N} = 0$  CFT theory with two marginal operators for massless scalars (pseudo-scalars) in  $\mathbf{1}_0$ ,  $\mathbf{15}_0$  and for the graviton  $\mathbf{8}_s$ , there is a 3d  $SU(4) \times U(1)$   $\mathcal{N} = 6$  SCFT theory and tow marginal operators associated with massless scalars (pseudo-scalars) are just in  $\mathbf{15}_0$  [11], [24].

Therefore, since our ansatz breaks supersymmetry obviously and that at least for  $k = 1$  the skew-whiffed solution with  $S^7$  is also supersymmetric, we assume that ansatz could not be applicable in the case, seemingly. For the case  $k = 2$  as well, for all garvitinos there is the maximal supersymmetry  $\mathcal{N} = 8$  in the bulk. Thus, it is also excluded on the fact that solution make difference between  $\mathbf{8}_{v,c}$  and  $\mathbf{8}_s$  and breaks all supersymmetries.

Anyway, the main question here is on uplift of the ten-dimensional ansatz (3.1) to a eleven-dimensional one. The best consistent ansatz may be

$$A_6 = (f \, d\varphi \wedge \omega \wedge J^2) \Rightarrow F_7 = df \wedge d\varphi \wedge \omega \wedge J^2 - f \, d\varphi \wedge J^3 \quad (3.15)$$

Inserting the ansatz into the eleven-dimensional form-field identity and equation

$$dF_4 = 0, \quad d *_{11} F_4 + \frac{1}{2} F_4 \wedge F_4 = 0 \quad (3.16)$$

we see the equation satisfies trivially while the identity satisfies with same scalar Laplace equation of (3.6) and also if

$$d *_7 (d\varphi \wedge \omega \wedge J^2) = 0. \quad (3.17)$$

which don't satisfy of course. However it doses not seem to create a serious problem because the original 7-form that couples to the electric M5-branes is satisfied and now its magnetic dual may be a partial solution and not an exact one. Again the ansatz and solution are  $SU(4) \times U(1)$ -invariant and so with discussions in this subsection, one may follow parallel lines with type IIA case.

However, we should mind again that for  $k = 1, 2$ , we don't have the founded mode in known 11d supergravity over  $AdS_4 \times S^7/Z_k$ . On the field theory side, also, the associated chiral operators are  $SO(8)_R$ -invariant while the gravity solution here is  $SO(6)_R$ -invariant. So, these two special cases don't mainly include in our discussions.

It is also notable that one could place  $e_{S^1/Z_k}^7 = \frac{1}{k} (d\varphi + k\omega) \equiv e_7$  instead of  $d\varphi$  in the ansatz (3.15). Now the added M5-brane probably wrap around  $S^6/Z_k \sim CP^2 \times S^1 \times S^1$ , in general.

## 4 Dual Boundary Solutions and Correspondence

### 4.1 Matching Bulk to Boundary

Here we find a dual description for the bulk pseudo-scalar mode based on AdS/CFT correspondence prescriptions [10], [23]. Following the discussions on the spectrums, we first note that our ansatz (3.1) is a singlet of  $SU(4) \times U(1)$ . That is because  $J$  and therefore  $\omega$  and also  $e_7$  are  $SU(4)$ -invariant as well as  $J$  and  $e_7$  don't carry any  $U(1)$  charge. So, the dual boundary operator should be a singlet of  $SU(4)_R \times U(1)_b$  as well (look at [7] too).

The next stage is what may be the dual boundary operator associated with this bulk state? First we note that turning on the normalizable mode  $\Delta_+$  is always considered as a different state in the same theory and not necessarily a deformation of the ordinal theory [14]. This next to some operators dual to such bulk states proposed in [15] and [16] make our task easier.

On the other hand, for a scalar in Euclidean  $AdS_4$  with Poincare upper half-plane coordinates, asymptotic behavior of the solution (3.6) near to the boundary at  $u = 0$  is

$$f(u, \vec{u}) \approx u^{\Delta_-} \alpha(\vec{u}) + u^{\Delta_+} \beta(\vec{u}) \quad (4.1)$$

where  $\Delta_{\mp} = 0, 3$ .  $\alpha$  and  $\beta$  have a holographic interpretation as *source* (the boundary value of the bulk field) and *one-point function* for the operator with conformal dimension of  $\Delta_+$ , respectively and vice versa for the operator of  $\Delta_-$ . Such a scalar can be quantized with either Dirichlet boundary condition  $\delta\alpha = 0$  (which can be used for any  $m^2$ ) or with Neumann boundary condition  $\delta\beta = 0$  (which can be used when the scalar masses are in the range  $-9/4 < m^2 L^2 < -5/4$ ). In the *usual* CFT [23], the source  $\alpha$  couples to an operator with  $\Delta_+$  (the normalizable mode).

Now, for the normalizable mode  $\Delta_+ = 3$  of our massless pseudo-scalar, with the solution (3.7) at hand, we can write

$$\alpha(\vec{u}) = f_0(\vec{u}), \quad \beta(\vec{u}) = \frac{c}{|\vec{u} - \vec{u}_0|^6} \equiv \frac{c}{r^6} \quad (4.2)$$

where  $c_2 = c$  and first term in (4.1) is dominated as  $u \rightarrow 0$ . Then, with the localized source  $f_0(\vec{u}_0) = \delta^3(\vec{u} - \vec{u}_0)$ , we have

$$\beta(\vec{u}) = \frac{1}{3} \langle \mathcal{O}_3(\vec{u}) \rangle_{\alpha} = -\frac{\delta W[\alpha]}{\delta \alpha} = \frac{\delta S_{on-shell}}{\delta \alpha(\vec{u})} \quad (4.3)$$

where  $\mathcal{O}_3$  stands for  $\Delta_+ = 3$  boundary operator,  $W$  is the field theory *generating functional* and  $S_{on-shell}$  is the bulk on-shell action. This means that with the pseudo-scalar bulk mode turned on, one should correct the boundary action as  $S \rightarrow S + W$  [25] with

$$W = -\frac{1}{3} \int d^3 \vec{u} \alpha(\vec{u}) \mathcal{O}_3(\vec{u}) \quad (4.4)$$

we should also note that here  $\alpha = c_1$  we set to 1.

## 4.2 The Boundary Solution

According to the previous mentioned arguments for the dual boundary operator  $\mathcal{O}_3$  and that it may have same structure of the ABJM lagrangian terms and also the proposed operators in [15], [16], [17], we employ the following operator

$$\mathcal{O}_3 = \text{tr}(Y_A^\dagger Y^A \psi^{B\dagger} \psi_B) \quad (4.5)$$

where the matter fields transform in same representation of  $SU(4)_R \times U(1)_b$  - invariant lagrangian (2.9), i.e. the scalars  $Y^A$  transform as  $\mathbf{4}_1$  and fermions  $\psi_A$  as  $\bar{\mathbf{4}}_{-1}$ .

Then, the matter is that as we had a  $U(1)$  neutral  $SU(4)$ -singlet pseudo-scalar mode in the bulk, Is this  $\mathcal{O}_3$  operator also singlet? Indeed if we take the matter fields in original representations, there is a singlet in  $\bar{\mathbf{4}} \otimes \mathbf{4} \otimes \bar{\mathbf{4}} \otimes \mathbf{4}$ . But we argued that the non-supersymmetric bulk mode agrees to a swapping of the representations  $\mathbf{s}$  and  $\mathbf{c}$  of the original ones (3.8) in ABJM. So the fermions can now sit in  $\mathbf{8}_s$  while the supersymmetry charges sit in  $\mathbf{8}_c$ . Let us take the singlet spinor field in  $\mathbf{8}_s = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{6}$  as one of  $\psi_B$ 's, say  $\psi_4 \equiv \psi$ , while  $Y^A$ 's are in the original representation of  $\mathbf{4}_1$ . With these representations, one can simply arrive in a  $SU(4) \times U(1)$ -singlet state from  $\mathbf{1} \otimes \mathbf{1} \otimes \bar{\mathbf{4}} \otimes \mathbf{4}$ .

Now by a looking at the field equations from the action (2.9), for simplicity and obtaining a right solution, we turn on just one scalar, say  $Y^4 \equiv Y$ . Next, we use the following ansatz

$$\psi_a^a = \frac{\delta_a^a}{N} \psi, \quad Y = h(r) I_{N \times N} \quad (4.6)$$

where  $h(r)$  a scalar function on the boundary,  $I_{N \times N}$  is the unitary-matrix and settings for the spinor field is same as that we used before [7]. So with last ansatz and settings for the matter fields, the potentials  $V_{bos}$  and  $V_{ferm}$  become zero. After that, the field equations from now the deformed action, read

$$D_k D^k Y + \frac{1}{3} \text{tr}(\bar{\psi} \psi) Y = 0 \quad (4.7)$$

$$i \gamma^k D_k \psi + \frac{1}{3} \text{tr}(Y^\dagger Y) \psi = 0 \quad (4.8)$$

$$\begin{aligned} \frac{ik}{4\pi} \varepsilon^{kij} F_{ij} - i[Y(D^k Y^\dagger) - (D^k Y)Y^\dagger] + \bar{\psi} \gamma^k \psi &= 0, \\ \frac{ik}{4\pi} \varepsilon^{kij} \hat{F}_{ij} - i[(D^k Y^\dagger)Y - Y^\dagger(D^k Y)] + \bar{\psi} \gamma^k \psi &= 0 \end{aligned} \quad (4.9)$$

note the  $i$  factor in front of the Chern-Simons term because of being in the Euclidean space. Now by taking  $Y^\dagger = Y$ , the second and third term in both (4.9) equations suppress. On the other hand, we should note that using the setting (4.6) is equivalent to considering just the  $U(1) \times U(1)$  part of the complete gauge group. Further, the fundamental matter fields of the ABJM are natural with respect to diagonal  $U(1)$  that couple to  $A_i^+$  of  $A_i^\pm \equiv (A_i \pm \hat{A}_i)$  whereas the orthogonal combination  $A_i^-$  acts as the baryonic symmetry.

Then, from (4.9), we can write

$$\begin{aligned}\frac{ik}{4\pi}\varepsilon^{kij}F_{ij}^+ &= -2\bar{\psi}\gamma^k\psi \\ F_{ij}^- &= 0\end{aligned}\tag{4.10}$$

Besides, to adjust with bulk we set  $A_i^- = 0$ .

Thereupon, one can simply see that conditions to satisfy (4.7) and (4.8) together are

$$\partial_k\partial^k h(r) = 0, \quad i\gamma^k\partial_k\psi = 0\tag{4.11}$$

Then, for the scalar and fermion we indeed use the solution and ansatz recently applied in [8] and [7], respectively. There are

$$\begin{aligned}h &= c_7 + \frac{c_8}{r}, \\ \psi &= \frac{(c_9 + i(x-x_0)^k\gamma_k)}{(c_9^2 + (\vec{u} - \vec{u}_0)^2)^\zeta} \eta\end{aligned}\tag{4.12}$$

where  $\eta$  is a arbitrary constant spinor. Putting the  $\psi$  ansatz into the relevant equation of (4.11) fix its form explicitly

$$\psi = \frac{\sqrt{N}}{2}i^{3/2}\sqrt{\frac{4}{5}}\frac{(x-x_0)^k\gamma_k}{((x-x_0)_k(x-x_0)^k)^{3/2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\tag{4.13}$$

Here  $c_9^\dagger = \frac{1}{2}i(x-x_0)_k\gamma^k$  and note that with Euclidean signature we have used  $\gamma^k = (\sigma_2, \sigma_1, \sigma_3)$  from (2.13).

Therefore, by using the field equations, the remaining and of course finite part of the action to compute is

$$S_{modi.} = -\int_{R^3} d^3r (\partial_i h)(\partial^i h) \Rightarrow S_{modi.}^{inst.} = -4\pi c_8\tag{4.14}$$

where to evaluate the value of the action, we have done same as [8], [2]. Indeed, we have used the clear solution of (4.12) with  $c_7 = 1$  and noted the contribution from  $r = 0$  is vanished.

Meanwhile, one can use the solution (4.13) with (4.10) to check the net magnetic charge of solution is zero namely

$$B^k = \frac{4\pi i}{3k}\bar{\psi}\gamma^k\psi \Rightarrow \Phi = \oint_s \vec{B}.d\vec{s} = 4\pi g = \oint_s F^+ = \oint_s \varepsilon^{kij}F_{ij}^+ ds_k = 0\tag{4.15}$$

where  $g$  and  $\Phi$  show the net magnetic charge and flux respectively;  $\vec{B}$  stands for the magnetic field and  $s$  is a round sphere at infinity. This certifies the  $U(1)$  invariance of the boundary solution to be identified with the bulk one.

As a substantial way for checking the correctness of dual solutions, we may write the correlation functions of the involved operator in the instanton background. Particularly, the leading contribution for the vacuum expectation value of the operator  $\mathcal{O}_3$  in background is

$$\text{tr}(Y^\dagger Y \bar{\psi} \psi) = \frac{\sqrt{4}}{5\sqrt{5}} \frac{c_8^2}{((\vec{u} - \vec{u}_0)^2)^3} \quad (4.16)$$

This is proportional with  $\beta(\vec{u})$ , which we gained from near to boundary analyzing of the bulk solution (3.7). One also can relate the constant coefficients with respect to (4.3). Altogether, we end the boundary and bulk calculations are consistent as expected.

## 5 Summary and Further Comments

In this note, we found a new instanton solution of the ABJM model as the best version of  $\text{AdS}_4/\text{CFT}_3$  correspondence. Indeed, we employed an ansatz for six-form field strength of ten-dimensional type IIA supergravity while the original backgrounds of the model keep unchanged. After satisfying the field equations and identities with ignoring most probably a small back-reaction on geometry, we arrive in a fully localized solution in bulk of Euclidean  $\text{AdS}_4$ . Because the solution appears as a result of D4 (M5)-brane wrapping around just the associated internal spaces, it is identified with a pseudo-scalar. This mode is already known in the spectrum of 10d supergravity on  $CP^3$  and 11d supergravity on  $S^7/Z_k$ , when the latter is considered as a  $U(1)$  Hopf-fibration on the former. It should of course be mentioned that for  $k = 1, 2$ , the original M2-branes, probe the flat space  $R^8$  and  $R^8/Z_2$  respectively and the situation here becomes somewhat obscure in that at least there is not such a bulk mode in known spectrums of 11d supergravity over  $\text{AdS}_4 \times S^7/Z_k$ .

On the other hand, this bulk mode exists when the supercharges are in the representation of  $\mathbf{8}_c$  in contrast to the ABJM supercharges, which are in  $\mathbf{8}_s$ . So, to connect the bulk to boundary we should switch the representations  $\mathbf{s}$  and  $\mathbf{c}$  of ABJM as the resultant theory is then for anti-membranes. Next, based on the state-operator correspondence, we found a proper boundary operator of the conformal dimension of  $\Delta_+ = 3$  matching to the bulk massless pseudo-scalar state. This boundary operator is  $SU(4)_R \times U(1)_b$ -singlet as the bulk ansatz is so. Afterwards, we see that to match with bulk solution, one should just keep a scalar and a fermion next to the  $U(1) \times U(1)$  part of the full gauge group. Then by deforming the action, while just the mentioned field are included, because of the operator and solving the boundary equations we find a finite action Euclidean solution on the boundary and see how the bulk and boundary solutions are mutually compatible.

In summary, we can tell that we have indeed added a anti-D4(M5)-brane to the 3d  $\mathcal{N} = 6$   $SU(4) \times U(1)$  M2(D2)-brane model of ABJM resulting in a 3d  $\mathcal{N} = 0$   $SU(4) \times U(1)$  anti-M2(D2)-brane theory, interestingly.

At the end, we comment briefly on some other maybe intersecting related points and issues. The first hint is about supersymmetry. In general the skew-whiffing procedure breaks

all supersymmetries expect when the internal space is  $S^7$  [11]. However, the rigid way to supersymmetry checking of ansatzs is to use the supersymmetry transformations of gravitinos. But the ansatz (3.1) and also (3.15) obviously break all supersymmetries. That is because the branes these fields couple to cover the internal spaces in all generality. In other words, the brane we add don't have the right *relative transverse directions* with branes in near horizon limit of ABJM to be known as a supersymmetric combination of branes according to well-known brane intersection rules [26]. It is also notable that magnetic dual of this added D4(M5)-brane is a D2(M2)-brane, whose some world-volume directions are in  $AdS_4$ . Looking into their behaviors and other related issues may be creditable too.

Nevertheless, one may consider some special arrangements of the associated internal spaces on which the branes wind. In other words, one may parameterize, for example, one  $CP^1$  with  $\theta_1, \varphi_1$  and another  $CP^1$  with  $\theta_2, \varphi_2$  besides fixing another coordinate, say  $\xi$ , to a constant value. Then, the remaining one  $\psi$  in (2.5) may be considered as a coordinate for  $S^1$ . When the five-dimensional world-volume of the added D4-brane in wrap on these  $CP^2 \times S^1$ , its effect appear as a point in lower four-dimensional theory. Its 11d counterpart according to (3.15) is a M5-brane that may wrap around now  $CP^2 \times S^1 \times S^1$ , where the sixth coordinate here is the U(1) fiber-bundle coordinate of  $\varphi$ .

The second hint is about stability. In general, supersymmetry ensures stability. Nevertheless, howbeit our solution is not supersymmetric but it is stable. Indeed, it is justified that all skew-whiffing solutions are stable at least perturbatively [27] and for  $S^7/Z_k$ , the stability is guaranteed for  $k \geq 2$  although a direct check of stability may be interesting.

The third hint is about the uplift of gravity solution to ten or eleven dimensional parent theories. Although it is known the Kaluza-Klein truncation on the fiber and lens spaces, of example those involved in ABJM, are consistent [28]; but for the special fields added it is a particular study. Handling this issue and uplifting the solution to higher dimensional theories next to trying to estimate the small back-reaction could be in order as well.

The last issue is about another way of matching the bulk to boundary instantons. Actually, in 10-dimensional type IIB supergravity over  $AdS_4 \times S^7$  versus 4-dimensional  $\mathcal{N} = 4$   $SU(N)$  Yang-Mills field theory, a similar bulk solution was adjusted with  $SU(2)$  gauge fields on the boundary. We address this issue for the current background in a forthcoming study.

## 6 Acknowledgements

The real science makes people reasonable and sensitive to the surrounding with logical behaviors. But unfortunately it seems some bodies don't know and really understand this. So, it seems I must thank! the Ilam University officials who are bothering me because I am seemingly not at their line. Nevertheless, I would like to thank my mother, father, brothers and sisters in home who encourage and help me to continue my studies, with a great interest.



# References

- [1] J. Maldacena, "*The large  $N$  limit of superconformal field theories and supergravity*", Adv. Theor. Math. Phys. 2, 231 (1998), [arXiv:hep-th/9711200].
- [2] G. W. Gibbons, M. B. Green and M. J. Perry, "*Instantons and seven-branes in type IIB superstring theory*", Phys. Lett. B 370, 37 (1996), [arXiv:hep-th/9511080].
- [3] M. B. Green and M. Gutperle, "*Effects of  $D$ -instantons*", Nucl. Phys. B 498, 195 (1997), [arXiv:hep-th/9701093].
- [4] M. Bianchi, M. Green, S. Kovacs, G. Rossi, "*Instantons in supersymmetric Yang-Mills and  $D$ -instantons in IIB superstring theory*", JHEP 9808, 013 (1998), [arXiv:hep-th/9807033].
- [5] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, " *$\mathcal{N}=6$  superconformal Chern-Simons matter theories,  $M2$ -branes and their gravity duals*", JHEP 0810, 091 (2008), [arXiv:0806.1218 [hep-th]].
- [6] K. Hosomichi, K. M. Lee, S. Lee, S. Lee, J. Park and P. Yi, "*A Nonperturbative test of  $M2$ -brane theory*", JHEP 0811, 058 (2008), [arXiv:0809.1771 [hep-th]].
- [7] A. Imaanpur and M. Naghdi, "*Dual Instantons in Anti-membranes theory*", Phys. Rev. D 83, 085025 (2011), [arXiv:1012.2554 [hep-th]].
- [8] M. Naghdi, "*A monopole Instanton-like effect in the ABJM model*", Int. J. Mod. Phys. A 26, 3259 (2011), [arXiv:1106.0907 [hep-th]].
- [9] A. Imaanpur, " *$U(1)$  Instantons on  $AdS_4$  and the uplift to exact supergravity solutions*", JHEP 1111, 041 (2011), [arXiv:1108.2786 [hep-th]].
- [10] E. Witten, "*Anti-de Sitter space and holography*", Adv. Theor. Math. Phys. 2, 253 (1998), [arXiv:hep-th/9802150].
- [11] B. E. W. Nilsson and C. N. Pope, "*Hopf fibration of eleven-dimensional supergravity*", Class. Quant. Grav. 1, 499 (1984).
- [12] S. Terashima, "*On  $M5$ -branes in  $\mathcal{N}=6$  membrane action*", JHEP 0808, 080 (2008), [arXiv:0807.0197 [hep-th]].
- [13] M. Benna, I. Klebanov, T. Klose and M. Smedback, "*Superconformal Chern-Simons theories and  $AdS_4/CFT_3$  correspondence*", JHEP 0809, 072 (2008), [arXiv:0806.1519].
- [14] V. Balasubramanian, P. Kraus and A. Lawrence, "*Bulk vs boundary dynamics in anti-de Sitter spacetime*", Phys. Rev. D 59, 046003 (1999), [arXiv:hep-th/9805171].
- [15] E. Halyo, "*Supergravity on  $AdS_{4/7} \times S^{7/4}$  and  $M$  branes*", JHEP 9804, 011 (1998), [arXiv:hep-th/9803077].

- [16] M. Bianchi, R. Poghossian and M. Samsonyan, "*Precision spectroscopy and higher spin symmetry in the ABJM model*", JHEP 1010, 021 (2010), [arXiv:1005.5307 [hep-th]].
- [17] D. Forcella and A. Zaffaroni, "*Non-supersymmetric CS-matter theories with known AdS duals*", Adv. High Energy Phys. 393645 (2011), [arXiv:1103.0648 [hep-th]].
- [18] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, "*A massive study of M2-brane proposals*", JHEP 0809, 113 (2008), [arXiv:0807.1074 [hep-th]].
- [19] K. Skenderis, "*Lecture notes on holographic renormalization*", Class. Quant. Grav. 19, 5849 (2002), [arXiv:hep-th/0209067].
- [20] C. S. Chu, P. M. Ho and Y. Y. Wu, "*D-instantons in AdS<sub>5</sub> and instantons in SYM<sub>4</sub>*", Nucl. Phys. B 541, 179 (1999), [arXiv:hep-th/9806103].
- [21] I. I. Kogan and G. Luzón, "*D-instantons on the boundary*", Nucl. Phys. B 539, 121 (1999), [arXiv:hep-th/9806197].
- [22] M. J. Duff, H. Lü and C. N. Pope, "*Supersymmetry without supersymmetry*", Phys. Lett. B 409, 136 (1997), [arXiv:hep-th/9704186].
- [23] I. R. Klebanov and E. Witten, "*AdS/CFT correspondence and symmetry breaking*", Nucl. Phys. B 556, 89 (1999), [arXiv:hep-th/9905104].
- [24] E. Halyo, "*Supergravity on AdS<sub>5/4</sub> × Hopf fibrations and conformal field theories*", Mod. Phys. Lett. A 15, 397 (2000), [arXiv:hep-th/9803193].
- [25] E. Witten, "*Multi-trace operators, boundary conditions, and AdS/CFT correspondence*", [arXiv:hep-th/0112258].
- [26] E. Bergshoeff, M. de Roo, E. Eyras, B. Janssen and J. P. van der Schaar, "*Multiple intersections of D-branes and M-branes*", Nucl. Phys. B 494, 119 (1997), [arXiv:hep-th/9612095].
- [27] M. J. Duff, B. E. W. Nilsson and C. N. Pope, "*The criterion for vacuum stability in Kaluza-Klein supergravity*", Phys. Lett. B 139, 154 (1984).
- [28] J. P. Gauntlett and O. Varela, "*Consistent Kaluza-Klein reductions for general supersymmetric AdS solutions*", Phys. Rev. D 76, 126007 (2007), [arXiv:0707.2315 [hep-th]].